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A NOTE ON THE DIMENSIONAL REGULARIZATION OF THE STANDARD MODEL COUPLED WITH QUANTUM GRAVITY

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Abstract

In flat space, γ_5 and the epsilon tensor break the dimensionally continued Lorentz symmetry, but propagators have fully Lorentz invariant denominators. When the Standard Model is coupled with quantum gravity γ_5 breaks the continued local Lorentz symmetry. I show how to deform the Einstein lagrangian and gauge-fix the residual local Lorentz symmetry so that the propagators of the graviton, the ghosts and the BRST auxiliary fields have fully Lorentz invariant denominators. This makes the calculation of Feynman diagrams more efficient.

The dimensional-regularization technique [1, 2] is the most efficient technique for the calculation of Feynman diagrams in quantum field theory. Its main virtue is that it is manifestly gauge invariant, when gauge bosons couple to fermions in a chiral invariant way. When gauge bosons couple to chiral currents, gauge anomalies can be generated. If the gauge anomalies vanish at one-loop, as in the Standard Model, then, by the Adler-Bardeen theorem [5], there exists a subtraction scheme where they vanish at each order of perturbation theory. This ensures internal consistency.

The definition of γ_5 in dimensional regularization [2, 3, 4] breaks the Lorentz symmetry in the dimensionally continued spacetime. The calculation of Feynman diagrams in parity violating theories is still efficient, because the continued Lorentz symmetry is not broken in the denominators of propagators, but only in vertices and numerators of propagators. Using appropriate projectors, a Feynman integral can be decomposed in a basis of scalar and fully Lorentz invariant integrals. The complication introduced by γ_5 is only algebraic and a computer can easily deal with it. Calculations have the same conceptual difficulty than in the parity invariant theories.

When the Standard Model is coupled with quantum gravity, γ_5 breaks the dimensionally continued *local* Lorentz symmetry. It is less obvious how to break the continued local Lorentz symmetry and maintain efficiency in the calculation of Feynman diagrams. In this paper I show how this can be done.

In the vielbein formalism the Einstein lagrangian

$$\mathcal{L} = \frac{1}{2\kappa^2}\sqrt{g}R \tag{1}$$

is more symmetric than the complete theory and must be supplemented with appropriate evanescent terms. It is natural to look for an arrangement of the regularization technique such that the denominators of propagators are fully Lorentz invariant. The symmetric gauge is not allowed, but a derivative gauge-fixing for the residual Lorentz symmetry, combined with a certain trick for the BRST auxiliary fields, do the job. The prescription of this paper works in arbitrary dimensions.

I work in the Euclidean framework. The conversion to the Minkowskian framework is straightforward. I denote the physical spacetime dimension with D and the continued dimension with $d = D - \varepsilon$. The Einstein action (1) is $SO(d)$ invariant, while the complete theory is only assumed to be $SO(D)\otimes SO(-\varepsilon)$ invariant. Since no confusion can arise, the Euclidean $SO(...)$ symmetries will be called "Lorentz" symmetries.

Before dealing with the $SO(d)$ breaking theories it is instructive to reformulate the regularization of pure gravity and gravity coupled with parity invariant matter. Here no $SO(d)$ breaking occurs, yet it is convenient to use a derivative gauge fixing for the Lorentz symmetry (rather than the symmetric gauge), because it admits a straightforward generalization to the case of gravity coupled with parity violating matter.

The vielbein is defined as usual in d dimensions. The curved space conventions for torsion, curvatures, covariant derivatives and connections are

$$
\begin{aligned}
\mathcal{D}e^a &= \mathrm{d}e^a - \omega^{ab}e^b = 0, \qquad\qquad \Gamma^\rho_{\mu\nu} = e^{\rho a}\partial_\mu e^a_\nu + \omega^{ab}_\mu e^a_\nu e^{\rho b}, \\
\omega^{ab}_\mu &= \frac{1}{2}\left(\partial_\mu e^a_\nu - \partial_\nu e^a_\mu\right)e^{\nu b} - \frac{1}{2}\left(\partial_\mu e^b_\nu - \partial_\nu e^b_\mu\right)e^{\nu a} + \frac{1}{2}g_{\mu\nu}\left(e^{\rho b}\partial_\rho e^{\nu a} - e^{\rho a}\partial_\rho e^{\nu b}\right), \\
R^{ab} &= \frac{1}{2}R^{ab}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}\omega^{ab} - \omega^{ac}\omega^{cb}, \qquad R^\mu{}_{\nu\rho\sigma} = \partial_\sigma\Gamma^\mu_{\nu\rho} - \partial_\rho\Gamma^\mu_{\nu\sigma} - \Gamma^\lambda_{\nu\sigma}\Gamma^\mu_{\lambda\rho} + \Gamma^\lambda_{\nu\rho}\Gamma^\mu_{\lambda\sigma}, \\
\mathcal{D}_\mu\psi_i &= \partial_\mu\psi_i - \frac{i}{4}\omega^{ab}_\mu\sigma^{ab}\psi_i + iA_\mu\psi_i + A^a_\mu T^a_{ij}\psi_j + ... \qquad (2)
\end{aligned}
$$

The Ricci tensor and the scalar curvature are defined as $R_{\mu\nu} = R^{ab}_{\mu\rho}e^{\rho b}e^a_\nu$, $R = R_{\mu\nu}g^{\mu\nu}$, where of course $g_{\mu\nu} = e^a_\mu e^a_\nu$. The BRST transformations are

$$
\begin{aligned}
se^a_\mu &= -e^a_\rho\partial_\mu C^\rho - C^\rho\partial_\rho e^a_\mu - C^{ab}e^b_\mu, \\
sC^\rho &= -C^\sigma\partial_\sigma C^\rho, \qquad s\overline{C}_\mu = B_\mu, \qquad sB_\mu = 0, \\
sC^{ab} &= -C^{ac}C^{cb} - C^\rho\partial_\rho C^{ab}, \qquad (3) \\
s\overline{C}^{ab} &= B^{ab} - C^{ac}\overline{C}^{cb} - C^{bc}\overline{C}^{ac} - C^\rho\partial_\rho\overline{C}^{ab}, \\
sB^{ab} &= -C^{ac}B^{cb} - C^{bc}B^{ac} - C^\rho\partial_\rho B^{ab}
\end{aligned}
$$

Here C^μ, $\overline{C}_\mu$, B_μ are the ghosts, antighosts and auxiliary fields of diffeomorphisms, while C^{ab}, $\overline{C}^{ab}$, B^{ab} are those of the $SO(d)$ local Lorentz symmetry.

Perturbation theory around flat space is defined as

$$e^a_\mu = \delta^a_\mu + \widetilde{\phi}^a_\mu. \qquad (4)$$

The matrix $\widetilde{\phi}$ is decomposed into its symmetric and antisymmetric components ϕ and ϕ', respectively:

$$\widetilde{\phi}_{ab} = \delta_{ac}\widetilde{\phi}^c_\mu\delta^\mu_b = \phi_{ab} + \phi'_{ab}.$$

Diffeomorphisms can be gauge-fixed with the common Lorentz-invariant gauge functions

$$\mathcal{G}^\mu \equiv \partial_\nu\left(\sqrt{g}g^{\mu\nu}\right). \qquad (5)$$

The gauge-fixing and ghost lagrangians are the BRST variation of

$$\overline{C}_\mu\left(\mathcal{G}^\mu - \frac{\lambda}{2}B_\mu\right).$$

Integrating the auxiliary field B_μ out, we find the familiar expressions

$$\mathcal{L}^{\mathrm{diff}}_{\mathrm{gf}} = \frac{1}{2\lambda}(\mathcal{G}^\mu)^2, \qquad \mathcal{L}^{\mathrm{diff}}_{\mathrm{ghost}} = \sqrt{g}\partial_\nu\overline{C}_\mu\left(\mathcal{D}^\mu C^\nu + \mathcal{D}^\nu C^\mu - g^{\mu\nu}\mathcal{D}_\alpha C^\alpha\right). \qquad (6)$$

Lorentz gauge-fixing for $SO(d)$ invariant theories. The most popular gauge-fixing of the Lorentz symmetry is $\phi'_{ab}=0$ (symmetric gauge). This is not very convenient for the generalization to parity violating matter. For reasons that will become clear later, I fix the Lorentz symmetry by means of the gauge-fixing functions

$$\mathcal{G}_L^{ab}=\mathcal{D}^\mu\omega_\mu^{ab}=\frac{1}{\sqrt{g}}\partial_\mu\left(\sqrt{g}g^{\mu\nu}\omega_\nu^{ab}\right). \tag{7}$$

These functions are scalars under diffeomorphisms, so the ghost lagrangian is already diagonalized. Observe that the gauge-fixing (7) is higher-derivative. To avoid propagators behaving like $1/p^4$ in the infrared, the "auxiliary" fields B^{ab} have to be inserted in an unconventional derivative way. This is legitimate, because the fields B^{ab} are anyway BRST-exact. Precisely, the gauge-fixing and ghost lagrangian of the Lorentz symmetry are the BRST variation of

$$-\sqrt{g}\overline{C}^{ab}\left(\frac{\xi}{2}\mathcal{D}^2B^{ab}+\mathcal{G}_L^{ab}\right),$$

where ξ is an arbitrary gauge-fixing parameter (that can be set to zero in the "Landau" gauge) and $\mathcal{D}^2=\mathcal{D}^\mu\mathcal{D}_\mu$:

$$\mathcal{L}_{\rm gf}^L=-\sqrt{g}\left(\frac{\xi}{2}B^{ab}\mathcal{D}^2B^{ab}+B^{ab}\mathcal{G}_L^{ab}\right),\qquad\qquad \mathcal{L}_{\rm ghost}^L=-\sqrt{g}\overline{C}^{ab}\mathcal{D}^\mu\partial_\mu C^{ab}. \tag{8}$$

In total, the gauge-fixed and ghost lagrangians are

$$\mathcal{L}_{\rm grav}=\mathcal{L}+\mathcal{L}_{\rm gf}^{\rm diff}+\mathcal{L}_{\rm gf}^L,\qquad \mathcal{L}_{\rm ghost}=\mathcal{L}_{\rm ghost}^{\rm diff}+\mathcal{L}_{\rm ghost}^L. \tag{9}$$

The ghost propagators equal the identity divided by p^2. The graviton propagators are

$$\begin{aligned}
\langle\phi_{\mu\nu}(p)\ \phi_{\rho\sigma}(-p)\rangle_0 &= \frac{\kappa^2}{2p^2}(\delta_{\mu\rho}\delta_{\nu\sigma}+\delta_{\mu\sigma}\delta_{\nu\rho})-\frac{\kappa^2\delta_{\mu\nu}\delta_{\rho\sigma}}{(d-2)p^2}+\\
&\quad+\frac{\lambda-2\kappa^2}{4p^4}(\delta_{\mu\rho}p_\nu p_\sigma+\delta_{\mu\sigma}p_\nu p_\rho+\delta_{\nu\rho}p_\mu p_\sigma+\delta_{\nu\sigma}p_\mu p_\rho),\\
\left\langle\phi_{\mu\nu}(p)\ \phi'_{\rho\sigma}(-p)\right\rangle_0 &= \frac{\lambda}{4p^4}(\delta_{\mu\rho}p_\nu p_\sigma-\delta_{\mu\sigma}p_\nu p_\rho+\delta_{\nu\rho}p_\mu p_\sigma-\delta_{\nu\sigma}p_\mu p_\rho),\\
\left\langle\phi'_{\mu\nu}(p)\ \phi'_{\rho\sigma}(-p)\right\rangle_0 &= -\frac{\xi}{2p^2}(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho})+\frac{\lambda}{4p^4}(\delta_{\mu\rho}p_\nu p_\sigma-\delta_{\mu\sigma}p_\nu p_\rho-\delta_{\nu\rho}p_\mu p_\sigma+\delta_{\nu\sigma}p_\mu p_\rho),\\
\langle\phi_{\mu\nu}(p)\ B_{\rho\sigma}(-p)\rangle_0 &= 0,\quad \left\langle\phi'_{\mu\nu}(p)\ B_{\rho\sigma}(-p)\right\rangle_0=\frac{1}{2p^2}(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho}),\quad \langle B_{\mu\nu}(p)\ B_{\rho\sigma}(-p)\rangle_0=0.
\end{aligned} \tag{10}$$

A more standard structure in (8), with non propagating auxiliary fields B^{ab}, namely

$$\mathcal{L}_{\rm gf}^L=-\sqrt{g}\left(\frac{\xi}{2}B^{ab}B^{ab}+B^{ab}\mathcal{G}_L^{ab}\right),$$

can be obtained from (8) formally replacing ξ with $\xi/\mathcal{D}^2$. The corresponding propagators are obtained replacing ξ with $-\xi/p^2$ in (10). Then, however, the first term of the new $\langle\phi'\ \phi'\rangle$ behaves

like $1/p^4$ in the infrared. This behavior generates annoying IR divergences in $D = 3$ and $D = 4$. The trick (8) is safer because it avoids this problem.

Now I prove that the formulation (8) admits an immediate generalization to $SO(d)$ breaking models.

The breaking of $SO(d)$ to $SO(D) \otimes SO(-\varepsilon)$: propagators with $SO(d)$ invariant denominators. Now I assume that the regularization preserves diffeomorphisms and the local Lorentz symmetry group $SO(D) \otimes SO(-\varepsilon)$. The Lorentz indices $a, b, c \ldots$ running from 1 to d are decomposed into physical Lorentz indices $\bar{a}, \bar{b}, \bar{c} \ldots$ running from 1 to D, and evanescent Lorentz indices $\hat{a}, \hat{b}, \hat{c} \ldots$ running from D to d (with D excluded): $a = (\bar{a}, \hat{a})$, $b = (\bar{b}, \hat{b})$, etc. The curvature tensor decomposes into

$$R^{\bar{a}\bar{b}} = \overline{R}^{\bar{a}\bar{b}} + \omega^{\bar{a}\hat{c}}\omega^{\bar{b}\hat{c}}, \qquad R^{\bar{a}\hat{b}} = D\omega^{\bar{a}\hat{b}}, \qquad R^{\hat{a}\hat{b}} = \hat{R}^{\hat{a}\hat{b}} + \omega^{\bar{c}\hat{a}}\omega^{\bar{c}\hat{b}},$$

etc., where $\overline{R}^{\bar{a}\bar{b}}$ and $\hat{R}^{\hat{a}\hat{b}}$ are the $SO(D)$ and $SO(-\varepsilon)$ curvatures, respectively, and D_μ denotes the $SO(D)\otimes SO(-\varepsilon)$ covariant derivative. Observe that $\omega^{\bar{a}\hat{b}}$ transforms as a tensor. Similarly, objects such as $\omega_\mu^{\bar{a}\hat{b}}\omega_\nu^{\bar{a}\hat{b}}g^{\mu\nu}$ are scalars. The vielbein can be used to define covariant D'Alembertians in the physical and evanescent portions of spacetime:

$$\overline{D^2} = e^{\bar{a}\mu}e^{\bar{a}\nu}D_\mu D_\nu, \qquad \widehat{D^2} = e^{\hat{a}\mu}e^{\hat{a}\nu}D_\mu D_\nu.$$

The BRST transformations split as follows

$$\begin{aligned} se_\mu^{\bar{a}} &= -e_\rho^{\bar{a}}\partial_\mu C^\rho - C^\rho\partial_\rho e_\mu^{\bar{a}} - C^{\bar{a}\bar{b}}e_\mu^{\bar{b}}, \qquad sC^{\bar{a}\bar{b}} = -C^{\bar{a}\bar{c}}C^{\bar{c}\bar{b}} - C^\rho\partial_\rho C^{\bar{a}\bar{b}}, \\ s\overline{C}^{\bar{a}\bar{b}} &= B^{\bar{a}\bar{b}} - C^{\bar{a}\bar{c}}\overline{C}^{\bar{c}\bar{b}} - C^{\bar{b}\bar{c}}\overline{C}^{\bar{a}\bar{c}} - C^\rho\partial_\rho\overline{C}^{\bar{a}\bar{b}}, \\ sB^{\bar{a}\bar{b}} &= -C^{\bar{a}\bar{c}}B^{\bar{c}\bar{b}} - C^{\bar{b}\bar{c}}B^{\bar{a}\bar{c}} - C^\rho\partial_\rho B^{\bar{a}\bar{b}}, \end{aligned} \tag{11}$$

plus analogous rules obtained replacing all barred indices with hatted indices. Here $C^{\bar{a}\bar{b}}$ are the ghosts of the physical $SO(D)$ Lorentz symmetry, $C^{\hat{a}\hat{b}}$ are the ghosts of the evanescent $SO(-\varepsilon)$ Lorentz symmetry and so on.

Since the symmetry $SO(d)$ is broken, it is not possible to choose a completely symmetric gauge for the fluctuation $\widetilde{\phi}_\mu^a$. The simplest generalization of the symmetric gauge,

$$\phi'_{\bar{a}\bar{b}} = 0, \qquad \phi'_{\hat{a}\hat{b}} = 0, \tag{12}$$

kills the antisymmetric parts of the $D \times D$ and $(-\varepsilon) \times (-\varepsilon)$ diagonal blocks of the matrix ϕ'. The components $\phi'_{\bar{a}\hat{b}}$ are unconstrained. Since the Einstein lagrangian (1) is independent of $\phi'_{\bar{a}\hat{b}}$, the propagator of $\phi'_{\bar{a}\hat{b}}$ should be provided by extra lagrangian terms, e.g. $\sqrt{g}\omega_\mu^{\bar{a}\hat{b}}\omega_\nu^{\bar{a}\hat{b}}g^{\mu\nu}$. Then, however, it is not easy to have $SO(d)$ invariant denominators. Instead, a generalization of the Lorentz gauge functions (7) does the job in a simple way.

The Lorentz gauge functions (7) are replaced with the reduced set of gauge functions

$$\mathcal{G}_{rL}^{\bar{a}\bar{b}} = D^{\mu}\omega_{\mu}^{\bar{a}\bar{b}}, \qquad \mathcal{G}_{rL}^{\hat{a}\hat{b}} = D^{\mu}\omega_{\mu}^{\hat{a}\hat{b}}. \tag{13}$$

The auxiliary fields and antighosts are correspondingly reduced to the blocks $\overline{C}^{\bar{a}\bar{b}}, B^{\bar{a}\bar{b}}$ and $\overline{C}^{\hat{a}\hat{b}}, B^{\hat{a}\hat{b}}$, so the gauge-fixing and ghost lagrangians in the Lorentz sector read

$$\widetilde{\mathcal{L}}_{\text{gf}}^{rL} = -\sqrt{g}\left(\frac{\overline{\xi}}{2}B^{\bar{a}\bar{b}}D^2B^{\bar{a}\bar{b}} + B^{\bar{a}\bar{b}}\mathcal{G}_{rL}^{\bar{a}\bar{b}} + \frac{\widehat{\xi}}{2}B^{\hat{a}\hat{b}}D^2B^{\hat{a}\hat{b}} + B^{\hat{a}\hat{b}}\mathcal{G}_{rL}^{\hat{a}\hat{b}}\right), \tag{14}$$

$$\widetilde{\mathcal{L}}_{\text{ghost}}^{rL} = -\sqrt{g}\overline{C}^{\bar{a}\bar{b}}D^{\mu}\partial_{\mu}C^{\bar{a}\bar{b}} - \sqrt{g}\overline{C}^{\hat{a}\hat{b}}D^{\mu}\partial_{\mu}C^{\hat{a}\hat{b}}, \tag{15}$$

where $D^2 = D^{\mu}D_{\mu}$. The total ghost lagrangian is

$$\widetilde{\mathcal{L}}_{\text{ghost}} = \mathcal{L}_{\text{ghost}}^{\text{diff}} + \widetilde{\mathcal{L}}_{\text{ghost}}^{rL}. \tag{16}$$

The ghost propagators are still the identity times $1/p^2$, so their denominators are $SO(d)$ invariant.

The sum of the Einstein lagrangian (1) plus the gauge-fixing terms $\mathcal{L}_{\text{gf}}^{\text{diff}}$ and $\widetilde{\mathcal{L}}_{\text{gf}}^{rL}$ of formulas (6) and (14), are still insufficient to give a propagator to $\phi'_{\bar{a}\hat{b}}$. For this purpose, introduce an evanescent tensor field $B^{\bar{a}\hat{b}}$ transforming as a scalar under diffeomorphisms and as a vector under both $SO(D)$ and $SO(-\varepsilon)$ rotations and add

$$\Delta\mathcal{L} = -2\sqrt{g}\left(\frac{\xi}{2}B^{\bar{a}\hat{b}}D^2B^{\bar{a}\hat{b}} + B^{\bar{a}\hat{b}}\mathcal{G}_{rL}^{\bar{a}\hat{b}}\right), \qquad \text{where } \mathcal{G}_{rL}^{\bar{a}\hat{b}} = D^{\mu}\omega_{\mu}^{\bar{a}\hat{b}}, \tag{17}$$

to the Einstein action (1). This addition is clearly a scalar density. In this way the matrices B^{ab} and $\mathcal{G}_{rL}^{ab}$ are fully reconstructed. The diagonal blocks $B^{\bar{a}\bar{b}}, B^{\hat{a}\hat{b}}$ are auxiliary fields for the Lorentz gauge-fixings, while the non-diagonal components $B^{\bar{a}\hat{b}}$ are extra evanescent fields used for regularization. Since (17) vanishes in the formal limit $\varepsilon \to 0$, $\Delta\mathcal{L}$ is truly a regularization term. Therefore, even if $B^{\bar{a}\hat{b}}$ is not BRST exact, its introduction does not change the physics.

Recapitulating, the total gauge-fixed lagrangian is

$$\widetilde{\mathcal{L}}_{\text{grav}} = \mathcal{L} + \Delta\mathcal{L} + \mathcal{L}_{\text{gf}}^{\text{diff}} + \widetilde{\mathcal{L}}_{\text{gf}}^{rL}. \tag{18}$$

Now, set for a moment the gauge-fixing parameters $\overline{\xi}$ and $\widehat{\xi}$ equal to ξ. Then the quadratic part of the lagrangian (18) coincides precisely with the quadratic part of $\mathcal{L}_{\text{grav}}$ in (9):

$$\mathcal{L}_{\text{grav}} = \mathcal{L} + \mathcal{L}_{\text{gf}}^{\text{diff}} + \mathcal{L}_{\text{gf}}^{L} = \mathcal{L} + \Delta\mathcal{L} + \mathcal{L}_{\text{gf}}^{\text{diff}} + \widetilde{\mathcal{L}}_{\text{gf}}^{rL} = \widetilde{\mathcal{L}}_{\text{grav}} \qquad \text{for } \xi = \overline{\xi} = \widehat{\xi}, \tag{19}$$

up to cubic terms, due to the different definitions of covariant derivatives. In this case the propagators of ϕ, ϕ' and B coincide with the ones of formula (10).

More generally, the propagators depend linearly on $\overline{\xi}$ and $\widehat{\xi}$, because these are gauge-fixing parameters. It is easy to prove by direct computation that when $\xi \neq \overline{\xi} \neq \widehat{\xi}$ the propagators (10) are unmodified except for $\left\langle \phi'_{\mu\nu}(p)\ \phi'_{\rho\sigma}(-p)\right\rangle_0$, which is corrected by the replacement

$$-\frac{\xi}{2p^2}(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\mu\sigma}\delta_{\nu\rho}) \rightarrow -\frac{\overline{\xi}}{2p^2}(\delta_{\bar{\mu}\bar{\rho}}\delta_{\bar{\nu}\bar{\sigma}}-\delta_{\bar{\mu}\bar{\sigma}}\delta_{\bar{\nu}\bar{\rho}}) - \frac{\widehat{\xi}}{2p^2}(\delta_{\hat{\mu}\hat{\rho}}\delta_{\hat{\nu}\hat{\sigma}}-\delta_{\hat{\mu}\hat{\sigma}}\delta_{\hat{\nu}\hat{\rho}}) + \\ -\frac{\xi}{2p^2}(\delta_{\bar{\mu}\bar{\rho}}\delta_{\hat{\nu}\hat{\sigma}}-\delta_{\bar{\mu}\bar{\sigma}}\delta_{\hat{\nu}\hat{\rho}}+\delta_{\hat{\mu}\hat{\rho}}\delta_{\bar{\nu}\bar{\sigma}}-\delta_{\hat{\mu}\hat{\sigma}}\delta_{\bar{\nu}\bar{\rho}}). \qquad (20)$$

I have therefore proved that all propagators have $SO(d)$ invariant denominators.

The regularization can be simplified if $SO(d)$ is broken just to $SO(D)$. Then the last two terms of (14) are moved from $\widetilde{\mathcal{L}}_{\mathrm{gf}}^{rL}$ to $\Delta\mathcal{L}$. The field $B^{\hat{a}\hat{b}}$ is interpreted as an extra evanescent tensor field, on the same footing as $B^{\bar{a}\hat{b}}$. The ghosts $\overline{C}^{\hat{a}\hat{b}}$, $C^{\hat{a}\hat{b}}$ and the last term of $\widetilde{\mathcal{L}}_{\mathrm{ghost}}^{rL}$ in (15) are suppressed. Finally, D_μ is interpreted as the $SO(D)$ covariant derivative.

Here I have used the second order formalism, but the arguments of this paper can be easily adapted to the first order formalism, where the Lorentz gauge-fixing (7) looks even more natural. In the first order formalism, gravity is described by the independent fields e_μ^a and ω_μ^{ab}. Relations (2) hold, except for the formula expressing ω_μ^{ab} in terms of e_μ^a. The BRST auxiliary fields and the ghosts are unchanged. The BRST transformations are (3) plus

$$s\omega_\mu^{ab} = -\omega_\rho^{ab}\partial_\mu C^\rho - C^\rho\partial_\rho\omega_\mu^{ab} - \mathcal{D}_\mu C^{ab}$$

(or the corresponding reduced versions, when the continued Lorentz symmetry is broken). The gauge-fixings and the ghost lagrangians are formally identical: it is sufficient to interpret ω_μ^{ab} as an independent field. When the continued Lorentz symmetry is broken, the evanescent extra tensor field(s) are the same ($B^{\bar{a}\hat{b}}$ and eventually $B^{\hat{a}\hat{b}}$) and the evanescent deformations $\Delta\mathcal{L}$ are formally identical to those of the second order formalism. However, the propagators become considerably more involved, since the quadratic part of the lagrangian is non-diagonal in the fields $\phi_{\mu\nu}$, $\phi'_{\mu\nu}$, ω_μ^{ab} and B^{ab}. The first order formalism is useful when the torsion is non-zero and, in particular, for the regularization of supergravity.

It is also immediate to generalize the arguments of this paper when the metric and the vielbein are expanded around generic curved backgrounds. This is useful for calculations with the background field method and in the presence of a cosmological constant.

A final comment concerns the stability of the action under renormalization. The action (18) produces nice propagators, but is obtained choosing the evanescent deformation $\Delta\mathcal{L}$ and the gauge-fixing in a very special way. It is natural to wonder if renormalization spoils this structure. Divergences can be subtracted in a RG invariant way only if every allowed lagrangian term is turned on, multiplied by an independent renormalized coupling that runs appropriately. So, to have complete RG invariance all types of evanescent deformations should be included

in $\Delta\mathcal{L}$ at the tree level, not just (17), but then the propagators do not have $SO(d)$ invariant denominators. This problem is avoided as follows. Known theorems [4, 6] ensure that the evanescent sector of the theory does not mix into the physical sector, but produces at most a scheme change. Therefore, evanescent counterterms can be subtracted just as they come, at higher orders, with no introducton of new independent parameters at the tree-level. If a special evanescent deformation, such as the $\Delta\mathcal{L}$ of (18), is used at the tree level, instead of the most general evanescent deformation, then RG invariance is violated in the renormalized correlation functions only by contributions that vanish in the physical limit $\varepsilon \to 0$. It is therefore possible to carry out every calculation with the propagators produced by (18) and (16). Observe that this argument is analogous to the argument commonly used for gauge-fixing parameters: if the gauge fixing-parameter ξ is not left free to run, but set to some special value, as in the Feynman and Landau gauges, then RG invariance is violated, but only in the BRST-exact sector of the theory.

Summarizing, for an efficient calculation of Feynman diagrams in the Standard Model coupled with quantum gravity using the dimensional regularization, the gravity sector can be regularized in the way just described and the matter sector can be regularized in the usual fashion.

The results of this paper are dimension-independent (for $D > 2$). In particular, they apply also to models such as three-dimensional Chern-Simons gauge theories coupled with two-component fermions and gravity [7]. At the theoretical level, they are useful for the study of consistent irrelevant deformations of renormalizable theories and the predictivity of certain classes of power-counting non-renormalizable theories, such as those studied in ref.s [8, 9]. At the phenomenological level, they are useful for calculations of gravitational radiative corrections in low-energy phenomenological models.

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